Eddy Flux of Heat and Momentum during Two Years at Stockholm-Bromma

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Abstract

The eddy flux of momentum, sensible heat and latent heat has been computed from measured temperature, humidity and wind data from the aerological station Bromma 59° 21′ N, 17° 57′ E. The results obtained have been compared with similar computations for Larkhill 51° 11′ N, 01° 48′ W made by Priestley. A comparison has also been made with values obtained by using geostrophic winds taken from synoptic charts. An estimation of the heat and momentum gained by eddy flux in the zone between 51° N and 59° N and north of 60° N has been made.

Introduction

The general atmospheric circulation is as yet not very well understood and in fact many of its features are not even known. Problems concerning the general circulation may be attacked from many points of view with the aid of different techniques. One method is to study turbulent disturbances of global magnitude, which appear in the atmosphere as moving cyclones, anticyclones, troughs and ridges. The basic theory underlying this attack is due to Reynolds who first studied the hydrodynamic forces which are set up by turbulent motion in a fluid or a gas. According to Reynolds a velocity vector $\mathbf{v}$ can be defined as the mean vector $\bar{\mathbf{v}}$ plus an additional vector $\mathbf{v}'$ such that $\mathbf{v} = \bar{\mathbf{v}} + \mathbf{v}'$ or in components $u_i = \bar{u}_i + u_i'$. The bar denotes mean values taken over a time interval. The mean value of u_i' is then $\overline{u_i'} = 0$. The turbulent stress is equal to the transport of u-momentum in the direction of the y-axis or v-momentum in the direction of the x-axis. W. Schmidt introduced this concept into meteorology and studied the vertical transport of momentum, heat, moisture and matter in the atmosphere. He introduced the concept "Austausch" (A) such that the flux of matter per unit of area (S) normal to the direction of the gradient (n) is $S = -\frac{A}{\varrho}\frac{\partial s}{\partial n}$, where s is the density of the matter. Prandtl defined A as $\overline{\varrho w' l'} = \varrho l^2 \left|\frac{\partial u}{\partial n}\right|$ where l is the mixing length obtained from analogy with molecular motion according to the kinetic gas theory.

These studies concerned primarily the vertical flux of momentum, heat or humidity in the atmosphere. A. DEFANT (1921) first described cyclones and anticyclones as disturbances in the general zonal circulation, analogous to turbulent eddies in the frictional surface layer of the atmosphere. Defant computed A for momentum and heat at different latitudes using surface observations and mean charts. ÅNGSTRÖM (1925) stressed the importance of the flux of latent heat and LETTAU

(1938) found new methods for computing A, all giving values about 10^7 or 10^8 in cgs units.

JEFFREYS (1926) developed the theory for the transport of momentum and showed that cyclones and anticyclones are necessary elements in the general circulation since, by such eddies only, eastward angular momentum can be transported from the trade wind areas, where such momentum is obtained from the earth by surface friction, to the zone of the westerlies, where the momentum by friction is given back to the earth. This theory was based on the assumption of geostrophic winds.

The development of radiosondes and radio wind instruments as well as the recent appearance of a fairly dense network of aerological stations enable us to compute the flux of momentum, heat and humidity also at upper levels. An extensive investigation into momentum and heat transport, based on synoptic charts and on the assumption of geostrophic winds, has been published by STARR and WIDGER (1949) and a similar study including in addition the flux of humidity is in progress at Stockholm. PRIESTLEY (1949) points out that it is feasible to study the eddy flux directly, without any special physical assumptions such as concerning mixing length or geostrophic wind approximations. The eddy flux depends upon the turbulent components of wind, temperature and humidity, and the only thing one needs is regular observations up to a certain level. Today there are a great many aerological stations, and it would seem feasible to compute the variations of this flux during one year as well as from year to year and to obtain the geographic distribution of the flux. Priestley has computed such data for the station Larkhill in southern England. We have thought it worth while to study these same factors in another place, and below we give the results of our computations of heat transport and zonal stress for the Swedish station Stockholm—Bromma (59° 21′ N, 17° 57′ E).

Data

The period studied is mainly the same as that used by Priestley (1946—1947). In view of the very small number of observations available during the first four months of 1946 we have, however, excluded these and in their place have used the corresponding months from 1948. Daily radiosoundings for 0600 G.M.T. and the levels 950 mb, 900 mb, 700 mb, 500 mb, 400 mb, 300 mb and 250 mb have been used. The material from these two years is quantitatively and also perhaps qualitatively not as good as the English material. Out of the number of radiowind soundings made during 1947 with the American radio direction finder type S.C.R. 658, 92 % reached the 500 mb level, 82 % the 400 mb level, only 63 % the 300 mb level and a much smaller percentage higher levels. Therefore we have considered it necessary to restrict the complete computations to the levels between 950—400 mb, and the discussion mainly deals with these levels. For the higher levels, however, some values have been computed which will be discussed in certain connections. When reports of wind, and less frequently of temperature, are missing, surface and upper-air charts drawn at the Swedish Meteorological and Hydrological Institute have been used to obtain interpolated or extrapolated values. Wind values in these cases are geostrophic winds.

Two different radiosonde types have been used; an American instrument during the period May 1—June 30 1946 and the Väisälä-type later on. Comparisons between these instruments show considerable differences between the types, but there are reasons to suppose that such differences, consisting of a mean difference and random differences, have no correlation with the advection and can therefore be neglected here.

Computation of heat flux

At first we shall briefly review and comment upon the method used by Priestley for computing the flux of heat in sensible and latent form.

a) Sensible heat.

Let ϱ represent the density, V the south component of the horizontal wind velocity and T the absolute temperature of a volume of damp air. If c_p is the specific heat at constant pressure $c_p \varrho V T$ is the amount of sensible heat carried poleward per unit area and unit time. Over a period the average flux per unit time across unit area of a surface fixed in the west-east and vertical plane is $c_p \varrho \overline{VT}$:

Integrating vertically the total flow per unit length of longitude is $c_p \int_0^\infty \overline{\varrho VT} dz$ or according to Priestley approximately $\frac{c_p}{g} \int_0^{\overline{p}_0} \overline{VT} dp$, where $\overline{p}_0$ is the mean surface pressure and g the gravitational acceleration. R being the radius of the earth and λ the longitude, the total flow across the latitude φ is

$$HF = \frac{c_p}{g} R \cos \varphi \int_0^{2\pi} d\lambda \int_0^{\overline{p}_0} \overline{VT} dp$$

This quantity is not obtainable from the observations made at one station. We must restrict our present discussion to the local flux

$$\frac{c_p}{g} \int_0^{\overline{p}_0} \overline{VT} dp.$$

Let $\overline{V}$ and $\overline{T}$ denote mean values at a fixed level over a period of time.

Then since $V = \overline{V} + V'$ and $T = \overline{T} + T'$ we have $\overline{VT} = \overline{\overline{V}\overline{T}} + \overline{V'T'} + \overline{\overline{V}T'} + \overline{V'\overline{T}}$ The last two terms being equal to zero by definition, $\overline{VT}$ is the sum of two terms $\overline{\overline{V}\overline{T}}$, the advective flux and $\overline{V'T'}$, the eddy flux. The time mean value $\overline{\overline{V}\overline{T}}$ is obviously equal to $\overline{V}\overline{T}$. At any particular locality $\overline{\overline{V}\overline{T}}$ may have a large value, positive or negative, depending on $\overline{V}$, but this is partly compensated in other regions of the same latitude circle where $\overline{V}$ has the opposite sign. On the average $\overline{\overline{V}\overline{T}}$ may have a mean value $\overline{\overline{\overline{V}\overline{T}}}$ over the latitude circle, but to compute that value we must determine the values for $\overline{V}$ and $\overline{T}$ at many points having the same latitude. As we have the values for one point only we shall neglect the advective flux and study only the eddy flux $\overline{V'T'}$. According to a later discussion by PRIESTLEY (1950) the mean eddy flux is of the same sign and magnitude as the mean advective flux at least for the latitude 30° S. The eddy flux is in general positive but must not be so at all levels for any one station. The mean value taken over a latitude circle at one level, or the vertically integrated flux at one station, should however be considerably greater than zero if a sufficiently long period is chosen.

b) Latent heat.

For the discussion of the latent heat flux we follow the same method of reasoning as for the sensible heat. If L is the latent heat of vaporisation and X the mixing ratio the local flux of latent heat is

$$\frac{L}{g} \int_0^{\overline{p}_0} \overline{VX} dp$$

We shall study only the eddy flux $\overline{V'X'}$. The variation of L and c_p with temperature and pressure has been neglected in the following computations. L is given the value 590 and c_p the value 0.24.

Computed values of eddy flux of sensible heat

In table I is given the northward eddy flux of sensible heat from May 1946 to April 1948 at the 950, 900, 700, 500 and 400 mb levels. The values are given for 12 two-monthly periods, and for the summer, winter and yearly means. Fig. 1 gives a picture of the variations at Bromma and at Larkhill. For Jan.—April 1948 we have inserted the Larkhill values for Jan.—April 1946 and for Jan.—April 1946 the Bromma values for 1948. We see that the flux of sensible heat at Bromma is positive, i.e. directed poleward at least up to the 400 mb level. At Larkhill, however, the flux is southward already in the 400 mb level. Fig. 2 shows the variation with height of the flux during winter and summer and for the year. The summer period includes May—October, the winter period November—April. The most outstanding difference in fig. 2 between the curves for Bromma and Larkhill is the variation with height during the winter season. The curve for Bromma shows a marked increase of flux up to about the 700 mb level and from there on a decrease. The flux over Larkhill decreases rather constantly up to and above the 400 mb level not only in the mean but during each separate two-monthly winter period.

In fig. 3 is shown the time variation of the total flux of sensible heat integrated for the layer up to 400 mb. Corresponding values for

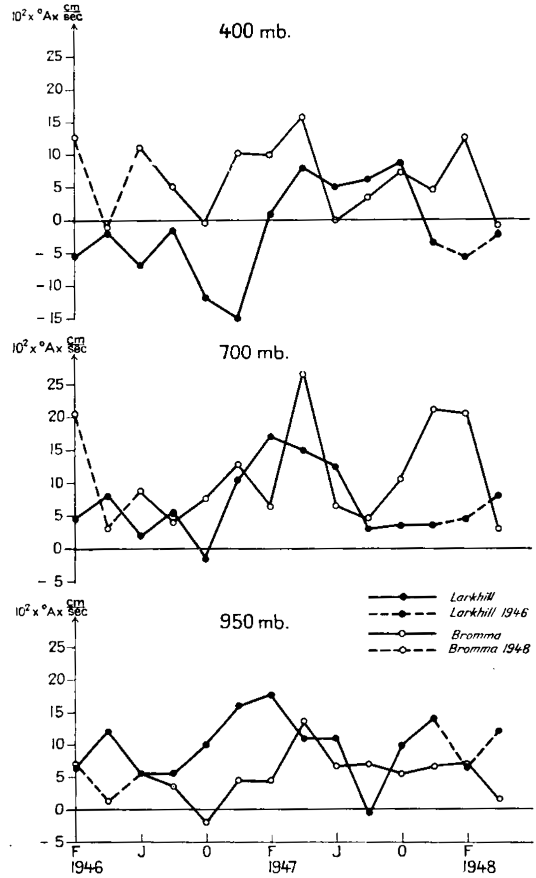

Fig. 1. Northward eddy flux of heat in sensible form $\overline{V'T'}$ at Bromma and at Larkhill at the 950, 700 and 400 mb levels.

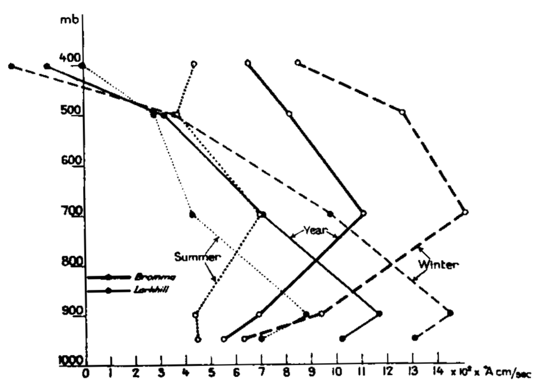

Fig. 2. Variation with pressure of eddy flux of heat in sensible form $\overline{V'T'}$ at Bromma and at Larkhill.

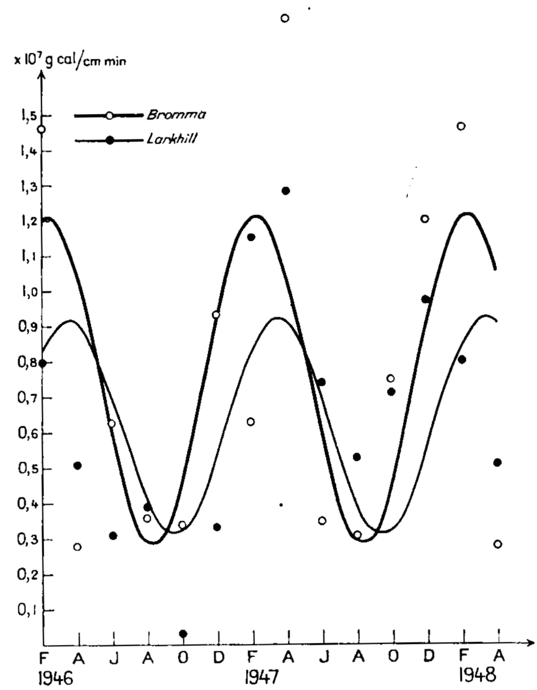

Fig. 3. Variation with time of total (vertically integrated) eddy flux of heat in sensible form at Bromma and at Larkhill.

Larkhill have been obtained by integration up to the highest levels. The flux over 110 mb was obtained by extrapolation. The values are distributed in a similar way. The curves drawn represent harmonic functions (y) of the annual variation of the flux in the form

$$y = A \cos (x + \alpha) + B$$

The parameters A, B and α have been found by the method of least squares. α is the time angle (0° or 360° for Jan. 1), A and B are constants, A being the annual amplitude and B the annual mean.

Priestley found for Larkhill

$$y = \left.\begin{matrix} [0.30 \cos (x - 50) + 0.65] \\ \sigma = 0.28 \end{matrix}\right\} \cdot 10^7 \text{ gcal/cm min}$$

For Bromma we obtained

$$y = \left.\begin{matrix} [0.46 \cos (x - 40) + 0.75] \\ \sigma = 0.33 \end{matrix}\right\} \cdot 10^7 \text{ gcal/cm min}$$

The annual mean value for Larkhill was obtained by integration up to the highest levels. If the integration is terminated at 400 mb, as in the Bromma case, the mean value for Larkhill becomes $0.63 \cdot 10^7$ gcal/cm min i.e. almost the same value as obtained previously by integration up to the highest levels, indicating that the integrated flux above 400 mb vanishes. The amplitude as well as the annual mean for Bromma is higher than that for Larkhill. Considering the rather large standard deviation (σ) of the values, this difference is not well established. The same can be said with regard to the phase difference which is only 10° while the values are given for intervals of two months. The lowest values are found in summer when both the meridional temperature gradient and the winds are weak.

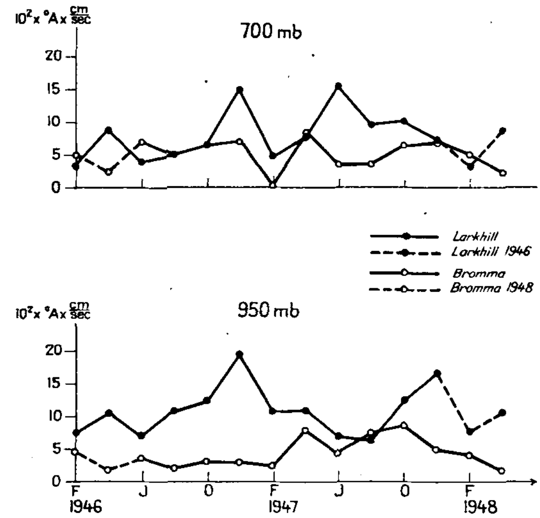

Fig. 4. Northward eddy flux of heat in latent form $\frac{L}{c_p}\overline{V'X'}$ at Bromma and at Larkhill at the 950 and 700 mb levels.

Computed values of eddy flux of latent heat

The mixing ratio normally decreases rapidly with height and as we shall see later, little of the heat transport will be lost if the computations are restricted to levels below 400 mb. In table II are given computed values of the northward eddy flux of latent heat. To facilitate a direct comparison with the values in table I the values $\frac{L}{c_p}\overline{V'X'}$ are given in table II. Fig. 4 shows the time variation of the flux in different levels, and fig. 5 shows the variation of the flux with height in winter and summer and for the yearly mean. We see that the flux of latent heat over Bromma is directed poleward at all levels up to 400 mb. It is of the same order of magnitude as the eddy flux of sensible heat. This must be heavily stressed as it is still assumed by some authors that this transport can be neglected. The importance of the latent heat flux is, however, much less at Bromma than at Larkhill, and it may be supposed to decrease even more at higher and cooler latitudes. From fig. 5 it appears reasonable to neglect the eddy flux of latent heat above the 400 mb level. From the same figure it is obvious that there is only a small difference in flux between the summer and winter seasons at both stations (see also the equations below). This must result from the fact that the values of mixing ratio reach maxima in the summer while migrating pressure systems with high values of turbulent winds reach their largest intensities in the winter. The values of the eddy flux of latent heat at Bomma and Larkhill for the period in question are represented by the following harmonic functions computed by the method of least squares;

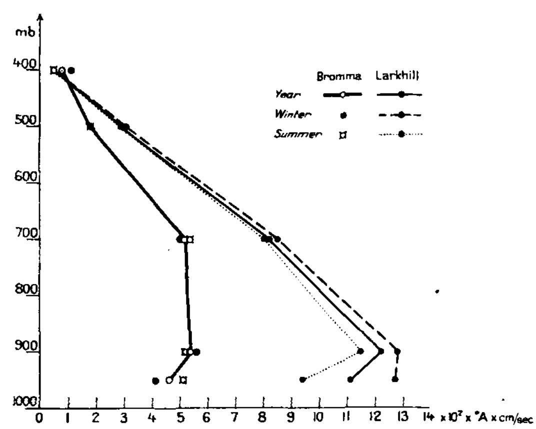

Fig. 5. Variation with pressure of eddy flux of heat in latent form $\frac{L}{c_p}\overline{V'X'}$ at Bromma and at Larkhill. For Bromma only the annual mean curve is drawn.

Table I. Northward eddy flux of sensible heat at Bromma

Quantity / unit	Level	1948 Jan. Febr.	1948 March April	1946 May June	1946 July Aug.	1946 Sept. Oct.	1946 Nov. Dec.	1947 Jan. Febr.	1947 March April	1947 May June	1947 July Aug.	1947 Sept. Oct.	1947 Nov. Dec.	Mean values Year	Mean values Summer	Mean values Winter
$\overline{V'T'}$ ($\times 10^2 \times {}^\circ A$ cm/sec)	950 mb	+ 7.1	+1.7	+ 5.6	+3.9	—1.9	+ 4.3	+ 4.3	+13.8	+6.9	+7.2	+ 5.4	+ 6.7	+ 5.4	+4.5	+ 6.3
	900 mb	+12.3	+5.5	+ 4.4	+2.0	+0.7	+ 9.7	+ 5.1	+14.3	+6.9	+4.5	+ 8.0	+ 9.6	+ 6.9	+4.4	+ 9.4
	700 mb	+20.6	+3.1	+ 8.8	+4.0	+7.5	+12.9	+ 6.6	+26.3	+6.6	+4.7	+10.3	+21.0	+11.1	+7.0	+15.1
	500 mb	+19.3	+2.6	+ 7.1	+5.8	+3.1	+10.5	+ 9.9	+22.1	—1.7	—0.2	+ 7.8	+11.5	+ 8.2	+3.7	+12.7
	400 mb	+12.2	—1.4	+11.4	+5.2	—0.7	+10.1	+ 9.9	+15.8	—0.2	+3.4	+ 7.1	+ 4.4	+ 6.5	+4.4	+ 8.5
$\times 10^7 \times$ gcal/cm, min	Total eddy flux	+ 1.46	+0.28	+ 0.63	+0.36	+0.34	+ 0.93	+ 0.63	+ 1.77	+0.35	+0.31	+ 0.75	+ 1.20	+ 0.75	+0.45	+ 1.05
	Mean pressure $\overline{p_0}$	1008	1012	1010	1008	1011	1014	1020	1003	1014	1012	1013	1002	1011	1011	1010

Table II. Northward eddy flux of heat in latent form at Bromma

Quantity / unit	Level	1948 Jan. Febr.	1948 March April	1946 May June	1946 July Aug.	1946 Sept. Oct.	1946 Nov. Dec.	1947 Jan. Febr.	1947 March April	1947 May June	1947 July Aug.	1947 Sept. Oct.	1947 Nov. Dec.	Mean values Year	Mean values Summer	Mean values Winter
$\frac{L}{c_p}\overline{V'X'}$ ($\times 10^2 \times {}^\circ A$ cm/sec)	950 mb	+ 4.3	+1.9	+ 3.8	+2.2	+3.1	+ 3.1	+ 2.3	+ 7.9	+4.7	+7.9	+ 9.0	+ 5.0	+ 4.6	+5.1	+ 4.1
	900 mb	+ 6.8	+2.2	+ 4.0	+4.0	+2.3	+ 8.5	+ 1.1	+ 8.4	+5.0	+5.9	+10.1	+ 6.6	+ 5.4	+5.2	+ 5.6
	700 mb	+ 5.1	+2.4	+ 7.3	+4.9	+6.1	+ 7.0	+ 0.2	+ 8.3	+3.4	+3.7	+ 6.5	+ 7.0	+ 5.2	+5.3	+ 5.0
	500 mb	+ 2.0	+0.5	+ 3.0	+1.7	+3.0	+ 2.5	+ 0.9	+ 3.2	—0.4	+1.3	+ 1.7	+ 2.3	+ 1.8	+1.7	+ 1.9
	400 mb	+ 1.2	—0.5	+ 1.7	+1.6	+1.6	+ 0.9	+ 0.2	+ 1.2	—0.3	+0.7	+ 1.5	— 0.1	+ 0.8	+1.1	+ 0.5
$\times 10^7$ gcal/cm, min	Total eddy flux	+ 0.38	+0.15	+ 0.43	+0.30	+0.34	+ 0.47	+ 0.07	+ 0.56	+0.23	+0.33	+ 0.51	+ 0.44	+ 0.36	+0.36	+ 0.35

Table III. Northward eddy flux of momentum at Bromma

Level	1948 Jan. Febr.	1948 March April	1946 May June	1946 July Aug.	1946 Sept. Oct.	1946 Nov. Dec.	1947 Jan. Febr.	1947 March April	1947 May June	1947 July Aug.	1947 Sept. Oct.	1947 Nov. Dec.	Mean values Year	Mean values Summer	Mean values Winter	
950 mb	+ 0.2	+ 0.8	— 0.1	+ 0.7	— 0.6	+ 0.1	+ 1.4	0	+ 0.5	+ 0.7	+ 1.3	— 0.1	+ 0.4	+ 0.4	+ 0.4	$\overline{U'V'}$ × 10⁵ cm²/sec²
900 mb	+ 0.6	+ 1.9	— 0.3	+ 1.3	— 0.5	— 0.5	+ 1.1	— 0.2	+ 0.4	+ 1.0	+ 1.7	+ 0.6	+ 0.6	+ 0.6	+ 0.6	
700 mb	+ 1.9	+ 1.9	— 0.8	+ 0.9	— 0.4	0	+ 1.5	0	— 1.8	+ 0.4	+ 2.5	+ 0.7	+ 0.6	+ 0.1	+ 1.0	
500 mb	0	+ 1.8	— 1.0	+ 0.5	— 2.4	+ 0.2	+ 0.7	— 1.1	— 2.9	+ 1.1	+ 3.0	+ 0.2	0	— 0.3	+ 0.3	
400 mb	— 1.0	+ 1.1	— 0.4	+ 2.2	— 4.3	+ 1.7	— 1.8	— 1.9	— 2.0	+ 1.3	+ 2.5	+ 0.6	— 0.2	— 0.1	— 0.2	
Reynolds' stress	+ 0.50	+ 1.06	— 0.40	+ 0.54	— 0.74	+ 0.04	+ 0.62	— 0.27	— 0.87	+ 0.50	+ 1.40	+ 0.29	+ 0.22	+ 0.09	+ 0.35	× 10⁸ dyne/cm

Table IV. Values of $\overline{V'T'}$, $\overline{V'X'}$ and $\overline{U'V'}$ computed from geostrophic winds 60° N and 15°—20° E (I) and measured winds at Bromma 59° 21′ N 17° 57′ E November 1947 (II)

(The units are the same as in Tables I—III)		$\overline{V'T'}$	$\overline{V'X'}$	$\overline{V'U'}$
I	850 mb	+ 39	+ 12	+ 156
II	900 mb	+ 40	+ 10	+ 95
I	700 mb	+ 54	+ 11	+ 120
II	700 mb	+ 45	+ 7	+ 99
I	500 mb	+ 3	+ 2	+ 144
II	500 mb	— 12	+ 2	— 110

Larkhill

$$y = [0.10 \cos (x + 30) + 0.64] \quad \sigma = 0.20 \Big\} \cdot 10^7 \text{ gcal/cm min}$$

Bromma

$$y = [0.06 \cos (x + 30) + 0.36] \quad \sigma = 0.14 \Big\} \cdot 10^7 \text{ gcal/cm min}$$

The amplitudes are very small and without statistical significance, whereas the mean value at Bromma is $0.36 \cdot 10^7$ gcal/cm min as compared with $0.64 \cdot 10^7$ gcal/cm min in Larkhill.

The total eddy flux

The sum of eddy flux in sensible and latent form is the total eddy flux. For Larkhill the total annual mean is $1.29 \cdot 10^7$ gcal/cm min. As we have shown previously the integrated eddy flux of sensible heat above the 400 mb level at Larkhill was very small. By analogy, if we neglect the eddy flux of sensible heat above 400 mb at Bromma we obtain a total flux for Bromma of $1.11 \cdot 10^7$ gcal/cm min. We may again point out that the flux of latent heat above the 400 mb level can certainly be neglected. The value of the total transport at Bromma is of course somewhat uncertain. However, a computation with the limited data available, of the transport of sensible heat for the period at the 300 mb and 250 mb levels, shows that the flux is slightly negative both at 300 mb and at 250 mb. An integration from 400 mb to 250 mb

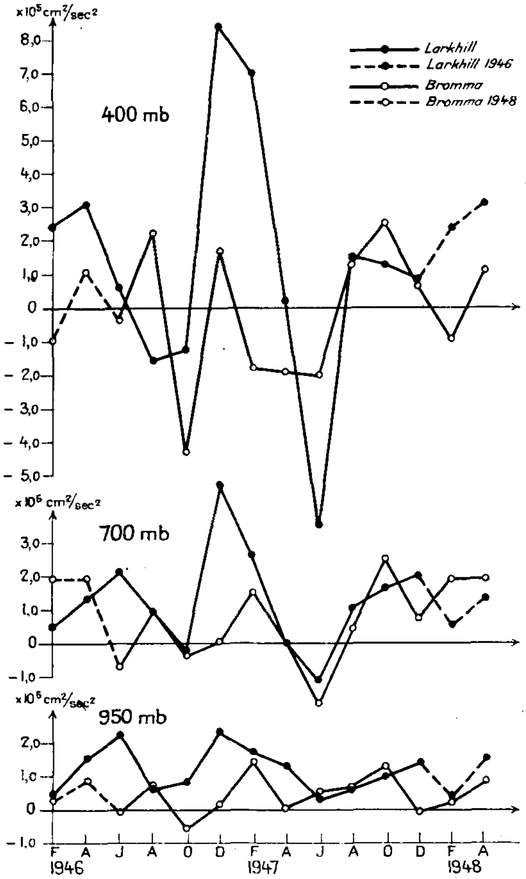

Fig. 6. Northward eddy flux of momentum $\overline{U'V'}$ at Bromma and at Larkhill at the 950, 700 and 400 mb levels.

does not give any contribution to the flux of sensible heat and in very high levels the contribution is, in all probability, very small.

Exchange of angular and linear momentum

According to Jeffreys the total poleward flow of angular momentum across a latitude circle in stationary conditions is proportional to

$$\int_0^{2\pi} d\lambda \int_0^{\infty} \varrho \overline{UV} dz$$

where U is the horizontal eastward wind component. Thus the total flow of angular momentum is equal to the flow of linear momentum. A computation of the eddy flux of momentum $\overline{U'V'}$ for Bromma has been carried our for the same 12 two-monthly periods as mentioned before. The results are given in table III (see also fig. 6). In the lowest levels the flux is in general positive, i.e. directed poleward, at higher levels there are both positive and negative values. Fig. 7 shows the summer, winter and annual means. The Bromma annual mean decreases slightly with height while the corresponding curve for Larkhill shows that it is fairly constant. The largest discrepancy between the two sets of values occurs in winter when the flux clearly increases with height at Larkhill and decreases slightly with height at Bromma. The total flow or stress per unit length can be represented by the harmonic curves;

Larkhill

$$y = \left.\begin{matrix} [1.78 \cos (x + 0) + 1.30] \\ \sigma = 1.70 \end{matrix}\right\} \cdot 10^8 \text{ dyne/cm}$$

Bromma

$$y = \left.\begin{matrix} [0.27 \cos (x - 20) + 0.22] \\ \sigma = 0.12 \end{matrix}\right\} \cdot 10^8 \text{ dyne/cm}$$

The zone south of Bromma thus exerts a stress towards the east upon the zone north of Bromma. This stress almost disappears during the summer. During the winter this stress is also relatively small, less than one sixth of the corresponding stress at Larkhill. At higher levels (300 mb and 250 mb) for Bromma the stress, computed from the scanty material available for these levels, is westward and therefore further reduces the absolute value of the total eastward stress. From WIDGER's (1949) data it appears that at 50° N the flux of momentum across the whole latitude circle in Jan. 1946 decreased with height from the 850 mb level to the 700 mb level and then increased to the 500 mb level. At 60° N the same conditions prevailed, but the absolute values were smaller, at the 700 mb level being almost equal to zero. Thus the eddy flux of momentum at Bromma as well as at Larkhill appears to be at variance with the mean transport across the corresponding latitude circles. It is not in any way surprising that single station values of $\overline{U'V'}$ are different from the mean of $\overline{UV}$, especially since a period of one month is fairly short. One might also imagine that these discrepancies depend to some extent upon the fact that Widger used geostrophic winds whereas for Bromma and Larkhill the actual winds were used. This question may be studied if the eddy flux for one station is computed from the actual winds and from the geostrophic winds as well. The transport of heat and momentum during November 1947 has been computed for the latitude circle 60° N (NYBERG and FRYKLUND, unpublished). If values for the longitudinal zone 15°—20° E are compared with values obtained from actual winds at Bromma 59° N 18° E, there is good agreement in the transport of latent heat (see table IV). The values obtained from the actual winds give smaller values however. The same is true of the sensible heat transport (note that at 500 mb the difference is fairly large and the flux is even of opposite direction). Finally, if we consider the eddy flux of momentum the discrepancies between the values obtained by the two methods are very large. However, the values for this month are small and too much significance should not be given to this discrepancy. If one assumes that the values reported by the stations are correct it is clear, however, that values of T and X may be accurately taken from synoptic charts but values obtained for the winds are less accurate. The winds were measured with a geostrophic wind rule. Both V' and U' (the eddy components of the geostrophic wind) contain inaccuracies and the correlation between them becomes uncertain. Therefore it is reasonable to suspect that although one may obtain a fairly good estimate of the eddy heat flux by using geostrophic winds it is not sufficient to use geostrophic winds obtained from synoptic charts for the computation of the eddy momentum flux for one station (or a small part of a latitude circle) during a period as short as one month.

The toroidal flux

Under stationary conditions we may write, according to Jeffreys,

$$\int_0^{2\pi} d\lambda \int_0^{\bar{p}_0} V dp = 0$$

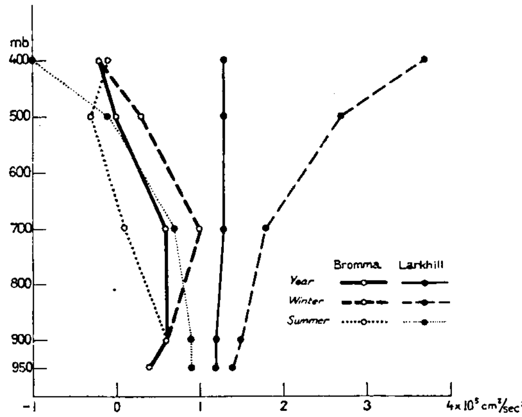

Fig. 7. Variation with pressure of eddy flux of momentum at Bromma and at Larkhill.

and assuming geostrophic winds, V_g, we have at every level

$$\int_0^{2\pi} V_g \, d\lambda = 0$$

If the mean of V at some levels has a net component (with mass transport) directed poleward or equatorward, i.e. if $\int_0^{2\pi} V d\lambda \neq 0$, as much mass must be transported in the other direction in other levels, assuming stationary conditions. ROSSBY (1941) represented the mean circulation of the middle latitude cell as toroidal, the mean value $\bar{V}$ being positive (poleward) at low levels due to the surface friction and negative at high levels. In view of the distribution of temperature and moisture with height, this would lead to a net transport of heat towards the north. The direction of the energy flux is doubtful as the potential temperature increases with height. Priestley in agreement with Rossby's hypothesis found such a toroidal component at Larkhill. A similar result was obtained at Bromma. We would like to suggest, however, that these results may be related to the geographical positions of the stations. Locally, one may get toroidal components even from geostrophic winds. A study of the variation of wind with height at the station Caribou (47° N, 68° W), showed a toroidal component in the opposite direction. It seems to us that one may not draw any conclusions concerning a general toroidal circulation from such material. Results recently published by PALMÉN (1950), RIEHL and YEH (1950) and PRIESTLEY (1950) show that at 40° and at lower latitudes there is an average poleward wind in the upper troposphere and an average equatorward wind in the lower troposphere. At 50° N these conditions according to INGRID REINEKE (1950) are presumably reversed. In the area over the British Isles and central Europe she has demonstrated a persistant deviation of the geostrophic wind towards the pole in lower levels and equatorwards in the upper troposphere. For higher latitudes no computations are known to us.

Net gain of heat and momentum in the latitude zone 51° 11′ N—59° 21′ N

It is tempting to compute the net eddy flow of sensible and latent heat supplied to the zone between 51° 11′ N and 59° 21′ N. This may be accomplished if we assume that the values for Larkhill and Bromma are representative for these latitudes. Such an assumption is certainly questionable but, considering the relatively long period used (2 years), it may be justified for the purpose of a first direct trial. It is hoped that other computations based on more complete material will be carried out.

If Q is the total eddy heat flow per second across the latitude φ in question we have

$$Q = 2\pi R \cos\varphi \cdot S$$

$$S = \left[\frac{c_p}{g}\int_0^{\bar{p}_0} \overline{V'T'}\,dp + \frac{L}{g}\int_0^{\bar{p}_0} \overline{V'X'}\,dp\right]$$

The mass of air between the latitudes φ_1 and φ_2 is

$$M = \frac{1}{g}\int_{\varphi_1}^{\varphi_2} 2\pi R^2 \cos\varphi \; d\varphi \int_0^{\bar{p}_0} dp$$

If the mean temperature increase per second in all levels and in the whole zone is equal to δT we get

$$Q_1 - Q_2 = c_p \cdot M \cdot \delta T$$

where Q_1 and Q_2 are the values of Q at the latitudes φ_1 and φ_2. When φ_1 is 51° 11′ and $\varphi_2 = 59° 21'$ we find the temperature increase in 24 hrs $\triangle T = 86400 \cdot \delta T = 0.30°$ C/24 hrs (corresponding to a heat gain of 0.05 gcal/cm² min in the zone). This means that in one day the eddy flux of total heat is sufficient to compensate for a heat loss by radiation of 0.3° C/24 hrs. No significant annual variation of this value has been obtained. Most of the heat gained seems to result from the condensation of water vapor in the zone.

In the same way we have computed the net gain of heat in the polar zone north of 59° 21′. We obtained the following values of $\triangle T$;

Annual mean	$\triangle T_m$	= 0.38° C/24 hrs
January	$\triangle T_{Ja}$	= 0.54° C/24 hrs
July	$\triangle T_{Ju}$	= 0.22° C/24 hrs

Most of the heat gain north of 59° N results from the transport of sensible heat.

The eddy flux of momentum across a latitude circle φ_i is

$$F_{m\varphi_i} = 2\pi R^2 \cos^2\varphi_i \cdot \frac{1}{g}\int_0^{\bar{p}_0} \overline{U'V'}\,dp$$

The net gain of momentum in a zone between the latitudes φ_1 and φ_2 is $F_{m\varphi_1} - F_{m\varphi_2}$. In a stationary state this must be compensated by the effect of surface friction.

If the mean surface frictional westward drag on the atmosphere in that zone is τ_0 dyne/cm² we have the total flux of eastward momentum from the atmosphere to the earth

$$F_{m\varphi_1} - F_{m\varphi_2} = \tau_0 \int_{\varphi_1}^{\varphi_2} 2\pi R^3 \cos^2\varphi \, d\varphi$$

From this equation we obtain the annual mean

$$\tau_0 = 1.5 \text{ dyne/cm}^2$$

In January the corresponding drag is 3.6 dyne/cm² and in July it is even slightly westward. Although the extreme values must be considered very uncertain owing to the great variability of the flux, the annual mean should give a much better estimate of the surface friction. However, even the annual mean has a standard error of 0.5 dyne/cm². According to the data the eddy flux of momentum would supply sufficient momentum to compensate for a large part of that lost in the zone due to surface friction. According to SVERDRUP (1945, p. 120) $\tau_0 = 1.5$ dyne/cm² corresponds to a surface wind over the ocean of 7 m/sec and 3.6 dyne/cm² corresponds to a wind of 10 m/sec.

The mean frictional drag of the earth north of 59° N is small. The annual value is 0.1 dyne/cm², the January value 0.2 dyne/cm² and the July value vanishes. These low values are consistent with the existence of both easterly and westerly mean winds in the polar zone.

Conclusions

The results for Bromma support the results obtained by Priestley for Larkhill. The eddy flux of sensible heat per unit length of latitude circle is approximately equal and has the same annual variation. The eddy flux of latent heat is at Bromma much less than at Larkhill, and the amplitude of the annual variation is very small. The eddy flux of momentum at Bromma disappears in the summer and even in the winter is less than 1/6 of the flux at Larkhill.

An estimation of the heat gain due to the eddy flux gives values for the mean temperature increase in 24 hours amounting to 0.3° C in the zone between 50°—60° N and 0.4° C north of 60° N. The temperature increase in 24 hours north of 60° N is 0.2° C in July and 0.5° C in January. The heat gain in the zone between 50° N and 60° N seems to be almost entirely dependent upon the eddy flux of latent heat, whereas north of 60° two thirds come from the eddy flux of sensible heat and one third comes from the eddy flux of latent heat. A similar estimation of the westward drag on the atmosphere due to surface friction gives a mean value of 1.5 dyne/cm² between 50° and 60° N. In January the drag is 3.6 dyne/cm² and in the summer the drag is even slightly westward. In the zone north of 60° N the mean drag even in the winter is a very small quantity.

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